Homework #4, PHY 674, 21 September 1995

(X16). Show that the map

$$\det : \mathrm{GL}\,(n,\mathbb{C}) \to \mathbb{C}^* = \mathbb{C}\setminus\{0\} \tag{16.1}$$
$$A \mapsto \det\ A \tag{16.2}$$

defines a homomorphism of groups. Is it injective, surjective, bijective ? Use this map to find a non-trivial l-dimensional complex representation of the group $\mathrm{GL}\,(n,\mathbb{C})$. Is this representation unitary ? For which l is it irreducible ? (4 points).

(X17). Let LG be a Lie algebra with Lie product $[-,-]$. Use bilinearity to show that $[x,x]=0$ for all $x\in LG$ implies $[x,y]=-[y,x]$ for all $x,y\in LG$. (4 points).

(X18). Show that any irreducible representation of an Abelian (finite) group G is one-dimensional. What does this tell us about the irreducible representations of cyclic groups ? Write down explicitly all irreducible representations and characters for the cyclic group of order n. (4 points) Hint: You may assume that the group is finite, if you wish.

(X19). Find all irreducible characters of the rotation-subgroup N of the equilateral triangle group $G=C_{3v}$. Since N is a normal subgroup, we can factor it out. Find the elements of the quotient group G/N and write down its multiplication table. Write down the irreducible representations of G/N. Find the elements of the direct-product group $N\times(G/N)$ and set up its multiplication table (This group is called C_{3h}). Show that the resulting group $N\times(G/N)$ is **not** isomorphic to G. Now, find all irreducible representations of the full equilateral triangle group G (no help from direct-product decomposition). (4 points)

(X20). Work out the multiplication table for the symmetry group of a general rectangle, divide the elements into classes and find all irreducible characters. (4 points)

Due Date:
Friday, 29 September 1995, 2 pm
in the green box in the physics department (or in class).

Dr. Stefan Zollner, A205 Physics, 294-7327, 21 September 1993.
Send questions to: `zollner@iastate.edu`.